Material Governance and Circular policies: how Waste Policies affect Household Appliances’ accumulation

by
Matteo Mazzarano
The CEntre for Research on Circular economy, Innovation and SMEs (CERCIS) is part of the Department of Economics and Management at University of Ferrara and, in particular, of the departmental development program funded by the Italian Ministry of Education, University and Research (MIUR) on a competitive basis for the period 2018–2022 to promote excellence in education and research (“Dipartimenti di Eccellenza”). The Department of Economics and Management is one of the 18 Departments funded in the Scientific Domain of Economics and Statistics at a national level.

Papers can be downloaded free of charge from the following websites: http://eco.unife.it/it/ricerca-imprese-territorio/centri-di-ricerca/cercis/working-papers

Enquiries: cercis@unife.it

CERCIS Working Paper
03/2021
By Matteo Mazzarano

Opinions expressed in this working paper do not necessarily reflect the position of CERCIS as a whole.
Material Governance and Circular policies: how Waste Policies affect Household Appliances’ accumulation

Matteo Mazzarano*
Università Cattolica del Sacro Cuore, Faculty of Mathematics and Physics

Abstract
Circular economy requires a great effort in monitoring material flows, market dynamics and policy makers perspective altogether. Here an attempt is made to understand the macro-economic drivers of accumulation and recycling. The application is made on Electrical and Electronic Equipment of household appliances. This category is relevant due to a specific effort of EU commission to tackle circular economy and material accumulation of critical raw materials. To understand the optimal dynamic of accumulation, innovation, and recycling, we developed a two-stage work. The former is a theoretical model of growth. This framework is used to delineate possible biases of a panel data model. To counter such problem, we applied a feasible generalized least square. Economies are interlinked and cross-sectional dependence in material accumulation persists. Variables used comprehend structural, socio-economic, and intrinsic characteristics of EEE stock of EU27+UK panel. As intrinsic, the panel uses average timespan, weight per capita and Shannon index of concentration. To control for breaks, the dummies for EU waste packages of 2008 and 2015 is used. The results show high significance among the general application and within each income class. Waste policies are strongly correlated to an increase of material accumulation, rather than a reduction. Socio-economic variables are generally significant, with evidence of decoupling. Lastly, timespan is positively correlated to material accumulation. The results highlight strong significance of policy effectiveness to the material dimension of economies, even at concluded economic cycles.

Keywords: Material stock; Ramsey model; Circular economy; Innovation; Waste policies; Electrical and Electronic Equipment

Abbreviations: Electrical and Electronic Equipment (EEE), feasible generalized least squares (FGLS), Environmental Kuznets Curve (EKC)

*matteo.mazzarano(at)unicatt.it
Introduction

Circular economy is an umbrella definition for the vision, policy framework and business models that disregards waste and use of virgin materials (Homrich et al. 2018). For the European Union, it is a policy nexus for the transition to a more sustainable society.Circularity is linked to different issues: sustainable development, trade security, economic growth and innovation. The environmental ones are related to the sustainability of our lifestyle. The strategic ones are related to the protection of our industrial assets despite external pressure of political instability and trade disputes. The former one refers to general better use of resources in a fair society should lead to a better tomorrow. The latter one touches the dependence of green and technological innovation to a set of strategic materials. Electrical and Electronic Equipment (EEE) represent a relevant class in both terms. For the sustainability of our system, they synthesize most functions for human welfare in a limited set of commodities. Therefore, keeping utility constant, electrification reduce material consumption (Pearson 2013). Since performance of such commodities is necessarily linked with material consumption of commodities. Circularity of EEE is therefore necessary for our purposes. At this point we need to cast a clear difference of what we define circular. A commodity that overcomes its role and is due to discard should be considered as a resource. Developing sectors that recover discarded EEE and their materials are not part of circular economy *per se* (Homrich et al. 2018).

The objectives of the European Union regarding EEE could be summarized according to the waste policy package in 2017 (European Parliament and Council 2017) and the communication on circularity “Closing the Loop” (European Commission 2015). The achievement of a more sustainable development will pass thorough a better use of current resources, improved recycling and use of secondary materials and finally circular innovation. While studies on innovation and sustainability have generated a prolific field of study, the impact to the recovery and accumulation of commodities is still relatively unknown at macro level. We will study therefore the impact of circular innovation to material accumulation of EEE. It is still unclear however if this affected the dimension of in-use stock of EEE. If products are discarded less, they are probably used more and more efficiently, increasing global efficiency. Recycling has been increases, and attention toward the recycling of waste of EEE has constantly increased globally.

We will divide the study in such manner. A concise literature review will shortly present three strands of literature. One relates the definition of anthroposphere and its material dimension. The other, is the theoretical models of materialistic relations within an economy. The latter relates the applied models that underpin hypothesis of economic cycles and endogeneity of material dependence. A methodological section take place further. It is split in a theoretical model and then an applied statistical one. The theoretical model has the objective to present a dynamical relation between innovation, recycling and economic growth. Its purpose in this study and future research is to link the applied model to the policy
framework, motivating the choice of variables and statistical tests. To introduce then the applied section we will present a data section. The results and the regression model are explained later in Results. We will explain the impact of such findings for material circularity in discussion. Lastly, the conclusion will summarize the innovation proposed by this work.

**Literature**

Our work uses notations and definition of different branches of resource economics modelling. We aim to link the exogeneity framework of inflow-driven mass equilibrium and economic modelling. The latter has used data estimated from the former methodology to assess decoupling and effectiveness of policy (Bringezu et al. 2004; Krausmann et al. 2009; Steger and Bleischwitz 2011; Wilts and O’Brien 2019). Inflow-driven assessment assumes that net addition to the economy are endogenous to new product acquisition and natural decay of stock (Fishman et al. 2014; Haberl et al. 2016; Wiedenhofer et al. 2019). This factor is assumed to vary according to time: as time departs from start, the proportion of substituted stock tends to one. When substitution factors converges to one, all net addition are devoted to substitution, hence material stock is considered mature (Bleischwitz et al. 2018; Winning et al. 2017). Despite common purpose, this approach is relatively rare to find cited among economics works. Nevertheless, we think it is relevant for material circularity modelling.

Policy making mainly relies on economic indexes such as Gross Domestic Product (GDP) in order to evaluate economic performance (Stiglitz, Fitoussi, and Durand 2018). This is relevant to evaluate ex post policy results, especially in mainstream environmental economics (see Environmental Kuznets Hypothesis). We believe the division Anthroposphere vs Geosphere nexus (Zoboli 2019) allows us to expand the number of scientific instruments. One is material mass of an economy (Matthews, Amann, and Bringezu 2000) and its circularity (Zotti and Bigano 2019). This is a valid alternative for circular economy studies as it pictures the material scales in a universally understanding. Weight does not inflate or deflate sensibly. A second one is the complexity of such stock. How it is diverse in relation to income, mass and new additions to an economy.

We will review circular economy models that are based on green growth. In such models, the planner maximizes utility as in others. Although beneficial, consumption leads to pollution which damage utility. As in the green Solow model, economic activity is used to increase welfare from consumption and financing pollution abatement (Brock and Taylor 2010; Stefanski 2010). Such approach is used to underpin Environmental Kuznets Curve (EKC). According to this approach, reduction of pollution appears in two phases: relative and then absolute decoupling. This literature affected the branch of circular economy, in the sense of recovering part of the pollutant/waste. Circularity as recycling modelling is divided in renewable and non-renewable approaches to growth model. Hotelling approaches are generally employed for a limited horizon (Cynthia Lin and Wagner 2007; Hotelling 1931). Material balance equation (Ayres 1995) is required to follow the second law (Bryant 2015;
Georgescu-Roegen 1971; Krysiak 2006). Growth model is generally used in a neoclassical context of utilitarianism (Di Vita 2001b, 2001a), in particular for tax effect on recycling of renewable resources (Di Vita 2004). In such cases, recycling increase the material extraction, inducing higher rates of economic growth. For non-renewable resources, there are similar results if perfect substitution of manufactured (or recycled) and natural capital is assumed (Di Vita 2007). When this condition does not apply, equilibrium is sustainable only for Hick’s neutrality i.e. innovation is unbiased (Comolli 2006). Application of circularity could be intended as pure materialistic models (without labour or innovation), where the main interest is to decouple waste, considered as pollution (George, Lin, and Chen 2015).

Innovation is used in defined horizon and infinite horizon. For the second one, introduction of recycling policy is strongly related to the level of consumption of exhaustible resources (Lafforgue and Rouge 2019). Innovation and knowledge level affect either pollution abatement (Chu and Lai 2014) or/and recycling rate (George, Lin, and Chen 2015). Despite our best effort, no level of technology might allow us to perfectly recover materials. The accumulation of useless materials could be resembled as a clear transliteration of the physical entropy (Kåberger and Månsson 2001; Kovalev 2016).

Considering primary and secondary materials as almost perfectly substituted, innovation in recycling works as in mining. In such case, increasing yield of extraction could be achieved even at lower ore grades (Stürmer and Schwerhoff 2012). Such assumption is relevant only when considering material composition of commodities, rather than homogeneous and linear products as assumed in literature. Previous studies focused on material composition commodities assuming mostly supply-side drivers, such as prices for aluminium in cars (Kandelaars and Van Dam 1998; Roberts 1992). We intend to apply a demand side approach from resource economics to test the possibility of demand side drivers in material composition of EEE. In order to explain the theoretical premise of our work we will hereby present a simple macro-economic growth model.

Methodology

Accumulation and circularity in theory

Our theoretical model is an inter-temporal optimization. A social planner chose a quantity of consumption “C”, new addition to material stock “N” and R&D circular innovation expenditures “F”. Retrofitting commodities for secondary uses (from recycling to recovery) could cost. For simplicity we will assume that circularity sector is endogenous to the economy and only net additions N costs to the system. In other words, social planner pays for net addition, technological development for recycling but not the cost of secondary flows. Such variable is collected in the sector of recovery \( R = gdM \). In this case, we will set unitary costs “p” to be constant for simplicity. In such economy, investments “I” are allocated to capital substitution \( \delta K \), F, capital variation and net material additions. They equal disposable income (equation 1).
\[ Y_t(R, M, K) - C_t = I = \dot{K}_t + F_t + \delta K_t + pN_t \] (1)

Expenditures in technology are proportionally accumulated into a stock of technological stock B in equation 2. We presumed that this rate of accumulation was deterministic and linear. The value of the parameter \( \eta \) determines the success of investment policies in generating the stock B: lower values of such parameter indicate greater efforts to be made for circular innovation. The purpose of this stock is to increase the recovery performance “g”:

\[ \dot{B}_t = \eta F_t \] (2)

Recovery performance is taken from climate change literature, as abatement technology cannot fully achieve perfect recovery. In this sense, the maximum potential of recovery is always lower than unity. In the previous study, we consistently found that this feature is present in EEE potential recovery, with the definition of the Artificial Ore Grade (Mazzarano 2020). This concept was chosen also to resemble the constant growth of entropy in environmental models. The accumulation of useless “work” could be trace in the cumulative distance to perfect recovery. We assumed that such limit is equal to “a” as explained in equation 3.

\[ g_t = a - e^{-bB} \rightarrow \lim_{B \to \infty} g = a \] (3)

Altogether with technological stock, economy accumulates a material stock “M”. Its variation is function of new additions and material decay. Despite the desire of material acquisition, stock M grows until maturity, where new additions are used only to substitute old units. Since treating waste generates hazardous materials to, all non-recycled commodities generate dis-utility according to \((1 - g)dM\).

\[ \dot{M}_t = N_t - dM_t \] (4)

\[ U = U[C, (1 - g)M] \]

\[ gdM = R \]

In this system, income is generated using three main inputs: capital K, material stock M, and recovered stock R. We assumed that R is equal to discarded material stock “dM” multiplied by recovery rate “g” as presented in group equation 4. In such way, it is possible to see that using two times affects general productivity. It could resemble either secondary material or recovered components. Since in this model intentionally overlooked market values, we ignore the differences. In terms of weight accounting, it does not differ. We rewrote production function as simply \( Y = f(K, M) \). Productivity is then composite of an exogenous factor and an endogenous one. To simplify at this stage calculation, we assumed “d” to be implicit, we will recall later how commodity durability affects macroeconomic equilibrium.
\[
Y_t = \varepsilon_t K_t^\alpha M_t^\beta (g_t dM_t)^{1-\alpha-\beta} = K_t^\alpha M_t^{1-\alpha} \left( g_t d\varepsilon_t^{\frac{1}{1-\alpha-\beta}} \right)^{1-\alpha-\beta} = \varepsilon_t K_t^\alpha M_t^{1-\alpha}
\] (5)

Our economy will have the purpose of increasing welfare by acquiring new commodities and ordinary consumption. The first value is relevant for accessing new functionalities, such as medical and personal utility. Despite the possible criticism over its venality, anthroposphere requires constant upkeep via new addition. As previously stated, such quantities go to substitute old stock and add new possibilities. Altogether, with recovery sector, productivity and income have overall increases. Our problem is therefore declined is such manner:

\[
\max_{C,N,F} W = \int U(C_t,N_t)e^{-\rho t} dt
\] (6)

\[
H_t = U_t e^{-\rho t} + \lambda_t (Y_t - C_t - F_t - pN_t - \delta K_t) + \mu_t \eta F_t + \nu_t (N_t - dM_t)
\]

The Hamiltonian \( H \) comprises the actualized utility function and each differential equation multiplied by the proper shadow prices. The first step to solve the problem is to derive control equations:

\[
\frac{\partial H_t}{\partial C_t} = 0 \rightarrow C_t = \frac{1}{\lambda_t e^{\rho t}}
\] (7)

\[
\frac{\partial H_t}{\partial N_t} = 0 \rightarrow p\lambda_t = \nu_t
\]

\[
\frac{\partial H_t}{\partial F_t} = 0 \rightarrow \lambda_t = \eta \mu_t
\]

Consumption equation is consistent with Ramsey–Cass–Koopmans model one. We tested similarly stability of system. Proportionality of capital and technological stock shadow prices offers some simplifications. Using conditions from equation 7.2 and 7.3, we can see that \( F \) is equal to capital change. As for production, we can see that the problem has two main differential equations: capital and material stock. We presented the mathematical passages within the appendix.

\[
\frac{\partial H_t}{\partial K_t} = \rho \lambda_t - \dot{\lambda}_t \rightarrow \dot{\lambda}_t = \lambda_0 e^{(\rho - \gamma_K + \delta)t}
\] (8)

\[
\frac{\partial H_t}{\partial B_t} \rightarrow U_B e^{-\rho t} + \eta \mu_t Y_B = \rho \mu_t - \dot{\mu}_t
\] (9)

\[
\frac{\partial H_t}{\partial M_t} \rightarrow \rho \nu_t - \dot{\nu}_t = \nu_t \left( \frac{1}{\rho} Y_M - d \right) + U_M e^{-\rho t}
\] (10)

\[
U_C = \lambda_t e^{\rho t}
\] (11)
The accumulation of commodities and satisfaction of other necessities reflects inter-temporal decision. We call elasticity of intertemporal substitution the responsiveness of consumption to the production factor returns. If we look at consumption, its responsiveness reflects market return of capital. The responsiveness of net addition reflects market return from material stock M minus generic capital return. Using the linear interaction of shadow prices, it is possible to see that stock of material flow and technology are necessarily proportional in long term. This allows for a strong reduction in complexity of the system, reducing the model to a two variables optimization.

\[
\frac{\dot{C}_t}{C_t} = \frac{1}{\sigma_c} (Y_K - \delta - 2\rho)
\]

\[
\dot{K} = \frac{1}{3 + d} (Y - C - \delta K)
\]

The solutions to the model should not differ drastically from the Solow-Swan outcome. The main particularity is that market interest rate is weighted by a discounting factor for spent materials. For our study, it is worth mentioning that both capital and therefore production are drivers of long-term stability of material accumulation.

\[
M = \frac{1}{dp} - \frac{e^{-d(K-K_0)}}{d}
\]

The steady state is therefore inversely proportional to the spent factor “d” and price “p” in equation (38). The positive relation with capital is matched by technological stock B, according to the efficiency “\eta”. We presented this relation in equation (39)

\[
M = \frac{1}{dp} - \frac{e^{\frac{d}{\eta}(B-B_0)}}{d}
\]

The modelling has outlined two operative hypotheses to estimate drivers of accumulation. The first one, relates the decoupling dynamics between economic growth and material accumulation. Whilst the economic cycle evolves according to a Ramsey–Cass–Koopmans model, material accumulation and technological change are partly redundant if the expenses to maintain them are linear. In this sense, is the interest rates contemplates the weight of spent commodities, theory of circularity works exactly are most theoretical premises of green growth. Therefore, an Environmental Kuznets Curve (EKC) setting will be outlined within the statistical model to test for tipping points in economic activity against material accumulation. Secondly, investments in circular economy are difficult to trace, by the stock of patents is a close representation of the variable B. Therefore, innovation stock could in theory affect material accumulation too.
While this dynamic optimization presented the theoretical premise of generic material accumulation, it cannot explain alone distribution of commodities within the stock. Instead of estimating another model with several commodities, we derived an equation from the equation (3.1).

**Stock Complexity**

Composition varies linearly with innovation/design and non-linearly with the economic cycle. This will be the hypothesis tested with statistical models. The current approach to test functional form around matter vs economy is the environmental Kuznets curve. Considering that, we assume parameters “a” and “b” to be unique for each material, heterogeneity might arise among results. Hence, it is possible that material composition of EEE will resemble decoupling for some and no relation with others.

Circular economy innovation affects indirectly the design of commodities. The economic cycle uses the budget to add new commodities and replace old ones to be recycled. Therefore, a rational planner will calibrate the stock composition to maximise its secondary material output. The relation between innovation stock and material stock should be then positive, indicating a positive value of circular productivity $\epsilon$. Indeed, this stock is a driver of material accumulation. Another aspect we can test is the relation between per-capita stock and stock complexity. Since it is difficult to understand the dynamics of functionalities brought by each new stock Kg of EEE, it is possible to assume that composition and weight have some form of interaction. The cycle could be addressed in terms of information theory and complexity. In theory, the maximum level of chaos stands between the minimum and the maximum probabilities of a set. In our case we do not talk of statistical variabilities but proportions within a basket of commodities. All identical commodities or perfect distribution of proportions represents the lowest levels of complexity. Complexity stands in the middle of these two possibilities. The relevance of last variable can be intuitively described as such. EEE resembles a diverse set of commodities with different composition matrixes. In case one commodity is predominant in weight, its materials will be defining in total EEE composition. Departing from the definition of stock variation, we can rewrite as in equation 16.

$$\sum_{vn} \frac{\dot{M}_{nt}}{M_{nt}} = \sum_{vn} \frac{N_{nt}}{M_{nt}} - \sum_{vn} d_{nt} \quad (16)$$

From this identity we continue with two passages. The first one is a simple transposition of the previous, keeping the signs:

$$\sum_{vn} \frac{\dot{M}_{nt}}{M_{nt}} - \sum_{vn} \frac{N_{nt}}{M_{nt}} = - \sum_{vn} d_{nt} \quad (17)$$

The second one applies natural logarithm to both terms as in equation 18.
\[
\sum_{\forall n} \ln \left( \frac{\dot{M}_{nt}}{M_{nt}} \right) = \sum_{\forall n} \frac{1}{\ln(d_{nt})} \sum_{\forall n} \ln \left( \frac{N_{nt}}{M_{nt}} \right)
\]

By multiplying both terms of last equation by the former one, we have:

\[
- \sum_{\forall n} d_{nt} \ln(d_{nt}) \sum_{\forall n} \ln \left( \frac{\dot{M}_{nt}}{M_{nt}} \right) = \left( \sum_{\forall n} \frac{\dot{M}_{nt}}{M_{nt}} - \sum_{\forall n} \frac{N_{nt}}{M_{nt}} \right) \sum_{\forall n} \ln \left( \frac{N_{nt}}{M_{nt}} \right)
\]

In both terms, we can spot the equations of entropy of durability \(S_d\) and entropy of relative acquisitions \(S_N\). The definition of entropy hereby used does not stem from the typical literature in environmental and ecological economics. We are referring to the information entropy used in information theory (Shannon 1948). By revising the equation with these definitions, we see that entropy of relative new addition is logistically related to stock growth:

\[
S_n = \left( S_d - \sum_{\forall n} d_{nt} \frac{\dot{M}_{nt}}{M_{nt}} \right) \sum_{\forall n} \ln \left( \frac{\dot{M}_{nt}}{M_{nt}} \right)
\]

The definition of entropy we used represents the concentrations of concentration. The higher its value, the less concentrated is the composition of commodities of EEE stock. Considering the functional form, stock growth is generally higher between the start point and the steady states. Low entropy, hence high concentration affects as previously stated the material composition of total stock. Therefore, stock maturity that is determined by economic cycle affects new addition concentration and then, material composition. The cycle could work in such way. First steps of accumulation are driven by novelty; thus, the set is increasingly complex. Once novelty is overcome, each new addition simply covers discovered functionalities. After this tipping point, complexity reduces. Linking this corollary with previous hypothesis, we need to prove that material distribution is a predictor of material composition, altogether with economic cycle and material stock.

To sum up the theoretical premise, economic system should select material composition according to endogenous variables of the anthroposphere. A society decides according to its income the amount of net additions \(N\), determining indirectly which should be the composition “\(c\)”, therefore “\(S\)”. Net additions cumulate until reaching a steady state. Since income affects necessarily stock and “\(S\)”, we need to test models separately to avoid endogeneity bias. Furthermore, aggregated timespan could be effective in determining distribution of commodities. It will be considered too.

**The econometric model**

The theoretical premise has some weak aspects that need to be tested. For instance, the hypothesis of closed economy would mean that cross-sectional dependence is null or negligible in a panel data. This should be untrue in our dataset, as we expect that within the EU panel, strong cross-sectional dependence
occurred. The estimation will require a strategy to counter such methodological limitation. The feasible generalized least squares (FGLS) trades sensitivity with robustness to biases in our theoretical model. In case estimated models will respond with significance, the strategy will repay the cost of sensitivity loss. This aspect is relevant due to the clustering of European Union. As we are going to see in next section, economic differences among nations induce high heterogeneity. Two approaches were used. One is clustering and the other is represented by variable choices. We added socio-economic factors to capture specific differences among nations. Physical variables are added to differentiate the intrinsic differences of EEE stock within clusters. We used patents for innovation stock.

\[ Y = X\beta + \gamma p + \theta age + \epsilon \]  

(21)

The impact of patent use in material-economy nexus has been studied before (Bringezu et al. 2004; Steger and Bleischwitz 2011). Since we expect non-linearities and decoupling dynamics due to literature results and theoretical model, we will apply the EKC Hypothesis to GDP. This will allow to search for tipping points. Since we expect similar dynamics with the composition, we will apply a similar formula to the Shannon index. We explained the statistical model in equation (21). Variable X refers to either GDP per capita as EKC (model 1), Shannon index as EKC (model 2) or Stock per capita (model 3). Variable p represents the cumulated sum of patents, as a knowledge stock for circular applications. We used these variables to control for “innovation” within this sector. As socio-economic variable, we used the percentage of population over the age of 65. This information is relevant, as elders tend to consume and replace less EEE, not using computers or IoT.

<table>
<thead>
<tr>
<th></th>
<th>Stock pro Capita</th>
<th>Shannon index</th>
<th>Average TimeSpan</th>
<th>GDP pro capita</th>
<th>Population Ratio over 65</th>
<th>Cumulated Patents</th>
<th>WEEEP</th>
<th>WFD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock pro Capita</td>
<td>1</td>
<td>0.755</td>
<td>0.226</td>
<td>0.814</td>
<td>0.286</td>
<td>0.316</td>
<td>0.277</td>
<td>0.384</td>
</tr>
<tr>
<td>Shannon index</td>
<td>0.755</td>
<td>1</td>
<td>0.153</td>
<td>0.684</td>
<td>0.09</td>
<td>0.143</td>
<td>0.045</td>
<td>0.217</td>
</tr>
<tr>
<td>Average TimeSpan</td>
<td>0.226</td>
<td>0.153</td>
<td>1</td>
<td>0.024</td>
<td>0.096</td>
<td>0.091</td>
<td>0.128</td>
<td>0.098</td>
</tr>
<tr>
<td>GDP pro capita</td>
<td>0.814</td>
<td>0.684</td>
<td>0.024</td>
<td>1</td>
<td>0.019</td>
<td>0.15</td>
<td>0.135</td>
<td>0.255</td>
</tr>
<tr>
<td>Population Ratio over 65</td>
<td>0.286</td>
<td>0.09</td>
<td>0.096</td>
<td>0.019</td>
<td>1</td>
<td>0.267</td>
<td>0.349</td>
<td>0.357</td>
</tr>
<tr>
<td>Cumulated Patents</td>
<td>0.316</td>
<td>0.143</td>
<td>0.091</td>
<td>0.15</td>
<td>0.267</td>
<td>1</td>
<td>0.04</td>
<td>0.042</td>
</tr>
<tr>
<td>WEEEP</td>
<td>0.277</td>
<td>0.045</td>
<td>0.128</td>
<td>0.135</td>
<td>0.349</td>
<td>0.04</td>
<td>1</td>
<td>0.577</td>
</tr>
<tr>
<td>WFD</td>
<td>0.384</td>
<td>0.217</td>
<td>0.098</td>
<td>0.255</td>
<td>0.357</td>
<td>0.042</td>
<td>0.577</td>
<td>1</td>
</tr>
</tbody>
</table>

*Table 1: Correlation table*
Data
Our work uses a collection of socio-physical data. Material variables comprise Stock per capita, Shannon index and average time span. They refer to the material component of our modelling. We took these data from the results of the ProSum project. The original dataset classifies the stock variation of EEE and its commodity components. Variation of the stock is defined as the sum of all net addition minus the discarded one. New addition is referred with ProdCom digits codes. Since the study of products lifespan has been made in UNU-key, all new addition is then aggregated within according to the latter key. Time span is then applied Shannon index is derived as explained in previous section from each product that define the EEE in-use stock. Time span is assumed to be constant per product category. To aggregate the value for the overall stock, we used a weighted mean according to pieces of each category. Classification according to Strata follows the same division of Prosum Project, where the first is considered the first tercile for economic activity of Europe and the third the last applicants to EU union.

1. Above 35,784 Euros: Austria, Netherland, Ireland, Sweden, Belgium, Denmark, Germany, United Kingdom, Finland, France, Luxemburg
2. Between 23.068 and 30.289 Euros: Spain, Slovenia, Cyprus, Czech Republic, Malta, Portugal, Slovakia, Italy, Greece

Since we witness this variability across income groups, we will apply the statistical model to the total sample and then repeat it for the three Strata. Among socio-economic variables, we collected Gross Domestic Product (GDP) pro capite, ratio of population over 65 years of age. Since this class of individuals tend on average to be less connected to IoT and recent addition to technology (Peacock and Künemund 2007), we assumed there could be some effect on EEE stock accumulation or composition.

To measure the effect of recycling innovation, we created a “patent stock” variable to capture the effect

Figure 1: Scar plot and smoothing process over average time span of EEE stock according to Stratum
that “B” might have on accumulation. We used the dataset from Eurostat for patents involved in circular economy and waste recycling. It is therefore a sectorial innovation stock. We chose it as a proxy for the technological evolution of circular sector. Its regressor will measure the impact that technological growth has on stock accumulation and composition. Finally, we added the two main policies that affected globally EU in terms of waste collection, classification of Waste framework package (WFP) and the WEEE package (WEEEP). They are dummy variable with 1 when they are implemented and 0 as the time before. We expect them to capture the structural changes that affected EEE accumulation in recent twenty years. Since these two variables are strictly time effects, we will not employ time dummies for our regressions.

Within table 1 we presented the correlation table. Socio-economic variables have strong correlation with physical variables as expected. Dummy variables are strongly correlated with population ratio, GDP pro capita and patent stock. To assess the impact of the endogeneity, we will use the correlation between residual and dependent value as rule of thumb. Since we need estimate the relation that policies have keeping constant socio-economic variables, we will focus on the other endogeneity issue. Patent stock is for instance correlated to population age ratio. This is interesting as we were expecting differently given the relation that investments and economic cycle have. Apparently, it is more probable to find cumulated patents in nations with higher quota of elder population.

I reported the summary of the variable in table 2. Looking at their distribution, material variables are leptokurtic. Narrower distributions with respect to socio-economic variables. Among the three, material stock is the most complex. The start value for most nation was around 80 Kg per capita, while in the end reached almost 300 Kg. The composition changed too. We multiplied Shannon index to 1000 in order to scope better the variation. It has a distribution with low standard deviation. Timespan, as previously noted, changed greatly between Strata.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Observations</th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Min</th>
<th>Pctl(25)</th>
<th>Pctl(75)</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock pro capita</td>
<td>448</td>
<td>186.306</td>
<td>61.463</td>
<td>49.509</td>
<td>140.694</td>
<td>236.817</td>
<td>308.45</td>
</tr>
<tr>
<td>Shannon index*1000</td>
<td>448</td>
<td>3121</td>
<td>15.7</td>
<td>2433</td>
<td>3038</td>
<td>3244</td>
<td>3369</td>
</tr>
<tr>
<td>Average Timespan</td>
<td>448</td>
<td>22.861</td>
<td>1.179</td>
<td>19.668</td>
<td>22.164</td>
<td>23.606</td>
<td>26.206</td>
</tr>
<tr>
<td>GDP pro capita</td>
<td>448</td>
<td>28288.37</td>
<td>20429.75</td>
<td>1613.867</td>
<td>13213.87</td>
<td>41561.37</td>
<td>120188.8</td>
</tr>
<tr>
<td>Population ratio</td>
<td>448</td>
<td>0.161</td>
<td>0.023</td>
<td>0.108</td>
<td>0.144</td>
<td>0.177</td>
<td>0.217</td>
</tr>
<tr>
<td>over 65</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cumulated Patents</td>
<td>448</td>
<td>11.004</td>
<td>21.151</td>
<td>0</td>
<td>0.59</td>
<td>12.283</td>
<td>141.66</td>
</tr>
<tr>
<td>WEEE package</td>
<td>448</td>
<td>0.25</td>
<td>0.433</td>
<td>0</td>
<td>0</td>
<td>0.2</td>
<td>1</td>
</tr>
<tr>
<td>WFD</td>
<td>448</td>
<td>0.5</td>
<td>0.501</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2: Summary Statistics
This definition reflects the distinction of economic activity within EU27+UK. The division is in three main groups. Nations above 35.784 Euros are considered first Stratum (Austria, Netherland, Ireland, Sweden, Belgium, Denmark, Germany, United Kingdom, Finland, France, and Luxemburg). Within the second, it is possible to find nations with a GDP per capita Between 23.068 and 30.289 Euros (Spain, Slovenia, Cyprus, Czech Republic, Malta, Portugal, Slovakia, Italy, and Greece). Finally, below 23.068 Euros lay nations of the third Stratum (Poland, Hungary, Estonia, Croatia, Lithuania, Latvia, Bulgaria, and Romania). In terms of lifespan, the weighted mean of the stock is 22 years. Composition in commodities changed its value between a minimum of 19.7 years rounded to 26 rounded. As it is possible to see in figure 1, this is a variable changing according to Stratum and time. It is probably due to the variation in composition and preponderance of certain categories of EEE. For instance, heavy domestic appliances have longer timespan than light ones. It is possible that in case of a reduction in average timespan, the economy is relatively accumulating lighter household appliances such as computers. GDP variability is widely noted and is clearly defined by the division expressed before. In fifteen years, it is possible to find Europeans with a GDP per capita of 1600 (Bulgaria 2000) to more than 120000 (Luxemburg 2015). Similar heterogeneity affects patents accumulation, as the maximum 141 is more than ten times the mean (11).

Overall, socio-economic heterogeneity requires clustering the application of the model. It is possible that each Stratum accumulated differently EEE during the period. Due to high correlation, we strategically chose to estimate models to avoid cross-selection issues. We choose to split the result in three cases, one for each class. Nevertheless, we will present first result according to EU27+UK in table 3.

Results

Across Eu27+UK, socio-economic variable largely explains both material variables under scope. Both policies seem to be effective in affecting composition and material accumulation. WEEEP was negatively affected composition, while WFD increased it. A possible explanation might be related to the objectives and timing of policies. According to chronological order, the former one was approved two years before the economic crash and the same year of Leman’ Brothers failure. Therefore, demand had still to adjust to the crisis. Or more importantly, since it is linearly correlated to stock, it is possible that the first break is registered before the peak. In general, the policy itself has imposed standards and waste reduction targets valid across Europe, determining a break in terms of waste policy overall. We find specific changes with the second policy. It refers to the waste categories of waste, practices, and producer responsibility. This might have affected collection and therefore, stock dynamics. We suggest interpreting results jointly. To summarize, waste policies are correlated across Europe with a reduction in terms of complexity of EEE stock, and reduction increase in terms of stock weight per capita. This outcome acquires general relevance when read with average timespan. Considering it is effective only
on composition, but not for total weight, it is possible that new additions have been neutral to the latter. Stock is a proxy for functionalities and the physical mass that embed them. In case stock is constant and shares change, it is possible that EU citizens acquired new functionalities keeping constant weight. The role of European policies has determined *ceteris paribus* an increase in terms of weight. This means that it slowed down the effect of substitution. Again, time-span effect is not statistically significant in all cases. In general, our hypothesis of material-technology cycle is not valid as patents and timespan are not significant for all *Strata* together. In-use stock registers a tipping point with the Shannon index. Furthermore, we register a possible decoupling mechanism for GDP and stock as in EKC. We will use this model as a yardstick to compare the *Strata* for their cycles.

**Stratum 1**

European countries with GDP per capita higher than 30000 euros register strong significance of economic cycle and of material cycle. For instance, time durability affects the distribution of commodities within the stock and the stock mass. The former negatively, while the latter positively, probably due to the longer timespan that only certain category of commodities have. Therefore, increased average commodity timespan is related to the increased importance of heavy equipment within First Stratum countries. Technological innovation in circularity positively affects material accumulation and negatively composition. Interestingly, WFD has had no effect on composition, keeping the same effect on stock as the whole panel EU27+UK. It probably means that it did not produce positive effect on total functionalities as for the rest of Europe. UKC supports both material settings and socio-economic cycle. This it means that material stock in rich countries has decoupled economic growth. Secondly, increases in mass have reached the saturation of functionalities. Elder population negatively affect composition. To summarize, material variables fit similarly to the socio-economic cycle. Stock complexity is saturated and economic cycle decouples by material accumulation. Policies and technological stock positively affect material accumulation.

**Stratum 2**

This stratum defines a wide category of citizen in EU. It is the middle tier category of income. In this case we find a weak presence of the material-technological cycle. For instance, circular patents stock has affected positively the EEE per pro capita, but not its composition. Timespan again has no effect on it however is strongly correlated with mass. Waste policies have had jointly a more conservative effect on mass/composition dynamics. Since policies have been less effective on composition but are generally effective on total weight, it is possible that this stratum is still in a transition phase. In this sense, socio-economic cycle affects seemingly material cycle as the previous models, but it is not the only factor. Intrinsic variables are less fitting and technological and policy variables too. Economic cycle and technological stock present similar estimates as the previous. Despite the weight of elder population such as in Italy, this variable in insignificant in determining the composition of EEE stock.
Stratum 3

Last result presents the outcome from estimation of our models for the third Stratum. With respect to the other models, material cycle models are relatively more fitting than the previous one. Cycles work similarly, but policies have been relatively less significant in affecting composition. Timespan, cumulative patents and elder population have no effect on composition, but rather on weight. Policies under consideration have had significant and great effect, as much in first Stratum. Interestingly, GDP has similar effects here as in previous models. Despite this continuity in results, the condition of less fitting is curious. We conclude that economic cycle is still immature to determine other variables.

According to our results, the hypothesis underlined by the theoretical model are overall correct. Distinction in control variables such as population dynamics and intrinsic variables are dependant to the Stratum. It is not surprising as these three groups are characterized by stark differences in EEE stock. Economies accumulate EEE commodities according to their needs, with weight and composition dynamics as strongly correlated. As they grow, both economic and material cycle concur for function accumulation. As decoupling incised, economic cycle mainly defines functionality accumulations, rather than gross weight. According to our preliminary results, waste policies have affected greatly tails of distribution, and have had lesser effect on middle tier economies. Results do not underpin overall the technological feedback from circularity patents.
Table 3: Results from the model estimation

<table>
<thead>
<tr>
<th>Stratum</th>
<th>EU27+UK</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shannon Index</td>
<td>Stock</td>
<td>Shannon Index</td>
<td>Stock</td>
<td>Shannon Index</td>
</tr>
<tr>
<td>Stock pro capite</td>
<td>4.839***</td>
<td>(0.440)</td>
<td>4.257***</td>
<td>(0.705)</td>
</tr>
<tr>
<td>Stock pro capite*2</td>
<td>-0.007***</td>
<td>(0.001)</td>
<td>-0.006***</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Population ratio over 65</td>
<td>-603.245***</td>
<td>(216.442)</td>
<td>-710.505**</td>
<td>(340.935)</td>
</tr>
<tr>
<td>Cumulative Patents</td>
<td>(216.442)</td>
<td>-0.559***</td>
<td>0.249***</td>
<td>(340.935)</td>
</tr>
<tr>
<td>WFD 2008</td>
<td>5.599</td>
<td>(0.228)</td>
<td>18.866*</td>
<td>(0.214)</td>
</tr>
<tr>
<td>GDP pro capite</td>
<td>0.013***</td>
<td>(11.503)</td>
<td>0.005***</td>
<td>(2.282)</td>
</tr>
<tr>
<td>GDP pro capite*2</td>
<td>-0.000***</td>
<td>(0.001)</td>
<td>-0.000***</td>
<td>(0.000)</td>
</tr>
<tr>
<td>Constant</td>
<td>2,771.404***</td>
<td>(92.035)</td>
<td>2,750.043***</td>
<td>(85.787)</td>
</tr>
<tr>
<td>Observations</td>
<td>448</td>
<td>28</td>
<td>448</td>
<td>28</td>
</tr>
<tr>
<td>Countries</td>
<td>28</td>
<td>0.491</td>
<td>28</td>
<td>0.490</td>
</tr>
<tr>
<td>R²</td>
<td>0.491</td>
<td>0.464</td>
<td>0.092</td>
<td>0.464</td>
</tr>
</tbody>
</table>

Standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1
Discussion

In this section we will elaborate the results to supply a proper narrative. The novelty of using material stock complexity underlines interesting points to material policy. Since consumers acquire different set of commodities, they indirectly choose their material too. Performances and materials are embedded within the commodity. Thus, market receive feedbacks on how to design commodities. Thus, it is probable that policies that aim to benefit consumers’ choices have indirect effect on material composition of commodities.

The policies considered are two. The first is the WEEEP. Approved for the creation of collection schemes where consumers return their WEEE free of charge. This directive aims to increase the recycling of WEEE and/or re-use\(^2\). The other policy is the WFD. The determination of a hierarchy was the policy breakthrough in waste management and prevention. The objective was to reduce as much as possible the volumes of waste produced. According to our results, both policies have induced increases in accumulated EEE. One possible explanation was that consumers started to reduce waste, therefore retaining some of the commodities. Since waste was reduced and net addition were seemingly constant over time, consumers kept accumulating. This is a positive outcome for secondary materials. Considering the rarity of CRM in EU, EEE represent a viable mine within our economies. Anthropospheric mining represent a valid resource for other stock, such as automotive and airborne sectors.

For policy making purposes, it is relevant to point out that our model does not distinguish between the market value of secondary materials of EEE and their recovered units. In other words, we assumed that the social planner somehow managed to control the cost of recovery within production cost. Another aspect is that the system tends to reduce the amount of irrecoverable mass (1-a). It is consistent with the policy implementation of WEEEP and WFD. The implementation of waste hierarchy defined the most undesirable flows. Such practice has reduced the production of total waste probably affecting total accumulation. Nations and firms abided accordingly redesigning products and collecting plans. Similarly, WEEEP ordered a framework for the collection cost-free for the consumer. Such practice has internalized the cost of collection within the prices. Therefore, policy dummies controlled for such aspects.

This paper linked economic cycle with stock dynamics and stock composition/complexity. According to our result, the composition indicator explained by the stock accumulation according to an inverted “U” functional form: after a tipping point total weight of EEE decouples stock complexity. We can express such relation in three main phases. At a start point, economic cycle is still at an immature stage, therefore net addition is neither complex nor heavy. After the accumulation of financial capital and innovation, the economy grows faster. Here we most probably find the peak in complexity due to the value that new functionalities have in this stage. Reaching the post-industrial stage, information is one of the main drivers of welfare. Cost of new additions is linear, but its returns are logistic. Therefore, at the frontier it is more difficult to increase dimension of stock

\(^2\) https://ec.europa.eu/environment/waste/weee/index_en.htm#:~:text=The%20new%20WEEE%20Directive%202012,into%20force%20in%20February%202003
weight and complexity. The former of the two is probably affected by the preference in terms of functionalities. Lighter and better performing commodities have probably higher requirements in material complexity; indeed, total weight is expected to be lower. Therefore, in terms of absolute dimensions, it is necessarily difficult to increase total manufactured stock weight at this stage. Furthermore, most of new additions are necessarily smaller and more complex. This is the main driver of new additions until substitution appears. Net addition tends to be zero. Hence, complexity of net addition should be lower at mature stage of economic cycle. These results are relevant for environmental economics in tow main ways. If it is possible to predict medium- and long-term new additions of EEE material stock, we can set objectives of material recycling based on real necessities.

Knowledge of recycling affects how much an economy dependent on foreign extraction and commodities production. Despite best effort, these activities are environmentally intensive (especially the former) and socio-economically intensive the latter (both). However, our study is useful to comprehend that if we consider the recycling sector as a source of raw materials or new commodities, circularity can partly reduce the dependency from external or virgin sources. The recycling sector has two main limits: how much commodities are wasted and how effective recycling technology is. Many authors call it the entropy problem. Secondly, if this model consistently predicts economic cycle and material cycle, we should expect higher pressure on mining related to EEE in next years. Nations entering the stage of development will require more material stock. This will add new functionalities and affect welfare, culture and democratization. Therefore, economic cycle might require higher environmental pressure in future years. In term of priorities, two main approaches of sustainability might assist. We could call one the nexus strong sustainability and lower information. It refers to the influence that precautionary principle should limit as much as possible mining activity and manufacturing outside circularity. This approach might induce in de-growth, reducing the number of functionalities. This would affect information. We think this vision might under-estimate the role that information and knowledge have in solving problems. The other is the weak sustainability with high information. In this setting, the main objective of policymaking is to increase the access to food, knowledge, and information access globally. The cost might be the increased environmental pressure. The relaxation of the precautionary principle is derived by the higher weight this vision gives to technology, knowledge and in general information. We believe that no matter the perspective, EEE issue relates both visions.

Conclusion

In this paper, an effort is made to define the theoretical dynamics between accumulation, economic cycle, and technological growth. We departed from a vanilla growth model using resource stock that increases over time. This economy reuses part of the waste produced for production. Material recovery from commodities could be assimilated to mining for social ecology. In order to model flows of matter, data from inflow-driven methodologies is recalled. The social planner improves the welfare by increasing consumption and new addition of material stock. This is employed as factor of production jointly with capital and recycled waste. Investment cycle drives research, new addition acquisition and capital accumulation. Research investment are
cumulated in a stock affecting recycling rate until its limit. Since waste is proportional to material stock, we simplified the production function as having two input variables. Productivity is therefore positively affected by each investment. The cost of recycling is null budget. The conclusion to the model were several. Material accumulation and complexity have a tipping point as EKC hypothesis, innovation stock affects material accumulation and socio-economic cycle decouple material accumulation. In this study, we linked them by giving a theoretical explanation and an applied model. We used as example EEE: they contain a wide variety of materials, there is a general concern for its waste and their acquisition signal the access to welfare. From our analysis, it appears that complexity of EEE category, economic growth and material stock are highly correlated among each other. We believe such phenomenon is related to material stock maturity. For endogenous growth, material stock tends to evolve and become more complex, heavier and it evolves according to economic growth. As the nexus reaches steady state, complexity reduces, converging to a constant value due to heterogeneous life cycle of commodities. Overall, the nexus considers two main tipping points from statistical results. One is the complexity and the other is the weight per capita. Accumulation of EEE positively affect complexity until an estimated tipping point, indicated by the inverted signs of the estimates. We interpreted this as the maturity of the stock in terms of acquired functionalities. As the stock reaches the maximum complexity, there are no new product to accumulate. Without novelty, the accumulation represents just new weight. Similarly, with mass per capita, economic cycle satisfy the EKC due to the different sings of estimates. Despite the intention to address differences between the various levels of economic growth of the Strata, estimated cycles are overall similar. This means that the estimated maturity for the nexus exist *ceteris paribus*. Controls varied without affecting the results of our main hypothesis. Our theoretical model was statistically consistent. Policy dummies proved to be consistent for waste effect but not overall significant for material complexity. We interpreted such result as a retention from consumers.

The paper offers a unique insight over the material stock-economic cycle nexus. In environmental economics, a theory of production and material recovery is defined. The dynamic between material complexity and stock growth is studied in model and estimation, presenting interesting insight for stock saturation studies. Finally, the relation with average product timespan and other EEE variables is presented. The application over the EEE allowed to take a look into the intersection with complexity and mass in a novel way. Complexity was intended to be a proxy of the functionality set that the EU resident could acquire. The difference with cited literature stands in the analysis object too. Most studies preferred to focus on “static” stock. Cited literature focused on material use, a sum of gravel raw materials. Inflow-driven works are strongly interested to the infrastructure dimension. Our insight to the complexity is less relevant to the monolithic aspects of static material stock. Sectors such as automotive and airborne services are composed by a variety of functionalities. This work could represent a valid starting point to evaluate maturity dynamics of those mobile stocks. We need to consider such aspect for a various set of reasons. They embed CRM such as REE and other Green materials, strategic for the carbon transition (Church and Crawford 2020). Furthermore, addressing secondary material potential and anthropogenic mining could temperate the risk of stranded asset in case of hard transition (Busch et al. 2014; Campiglio et al. 2018; Thomä and Chenet 2017). Proper assessment of material flows and its recycling
potential could present relevant element for circular economy. In its pure material point, it has positive benefits for climate change. Mobile and immobile stocks could be reconverted using material recovery by an advanced recycling sector. Outside dynamics of long term, management of secondary materials has major benefits for market security. Some of the CRM are located in few key exporters, such as the Popular Republic of China, Democratic Republic of Congo and Russia (ERECON 2014). Our material stock is approaching the state of maturity; hence every addition is mainly substituting old equipment. Circular management could hinder the dependency from external and sometimes volatile exporters.

Appendix: Capital growth to technological growth

The assumption of linear accumulation of R&D in a technological stock B implicates the proportionality of shadow prices. Recalling their co-state equations, we see:

\[
\frac{\partial H_t}{\partial K_t} = \rho \lambda_t - \dot{\lambda}_t
\]

\[
\frac{\partial H_t}{\partial B_t} = \rho \mu_t - \dot{\mu}_t
\]

If proportionality factor “\( \eta \)” is independent to time, we can derive by time the control equation for \( F \) without changing linearity condition. We changed signs with time derivative of both terms of the identity. We multiplied both terms of the original identity by the discount factor “\( \rho \)”.

\[-\dot{\lambda}_t = -\eta \dot{\mu}_t\]

\[\rho \lambda_t = \rho \eta \mu_t\]

By taking the vertical sum of the equation elements, we could trivially prove that co-state equations are proportional for “\( \eta \)”:

\[
\frac{\partial H_t}{\partial K_t} = \eta \frac{\partial H_t}{\partial B_t}
\]

At this point, we inverted the terms leaving the proportionality factor on the right and multiplied both by time derivative of the Hamiltonian function \( H \):

\[
\left(\frac{\partial H_t}{\partial K_t}\right)^{-1} \frac{\partial H_t}{\partial t} = \eta \left(\frac{\partial H_t}{\partial B_t}\right)^{-1} \frac{\partial H_t}{\partial t}
\]

The resulting equation represents a transformation of technological growth. B and K growth are proportional. Thus, by substituting for technological growth equation, we can say that \( F \) is equal to K growth:

\[\dot{B}_t = \eta \dot{K}_t = \eta F_t \rightarrow \dot{K}_t F_t\]

Appendix 2: Equilibrium and stability
Our system involves three control variables and three state equations. Since specific research capital B is linearly explained by generic capital K, we can simplify the analysis to just one system. It to capital vs consumption as explained in equations 20 and 21. It reveals a mainstream solution of saddle point. We constructed the Jacobian $J^1$ for the equations in such way:

$$J^1(\dot{K}, \dot{C}) = \begin{pmatrix} r & -1 \\ \frac{3 + d}{\sigma_C} & \bar{Y}_{KK} \\ \frac{1}{\sigma_C} & 0 \end{pmatrix}$$

Solution are very similar to Ramsey–Cass–Koopmans model. Since at steady state consumption growth is null, $J_{11}$ is zero. For production function concave to capital, determinant is negative. Since its eigenvalues have opposite sign, the system is a stable saddle point according to the manifold theorem.

A caveat is required. Rate of decaying (and therefore substitution) “d” is influenced by goods durability. According to inflow-driven methodology, it can be estimated to follow a CDF of a Weibull. Its independent variable is time of use. When a commodity reaches its natural limit, “d” should tend to one. Such interpretation of flows is used for data estimation and generates some complexities when modelling. Optimal condition would require that $(1-d)v$ tends to zero in long term. It is difficult to accept, since shadow price “$v$” would grow exponentially, since M would be rigid to N at $t^*$. In other words, since 1-d is zero after the “expiring date”, whatever choice of N would not change M. Therefore, M shadow price would increase; growth rate of “$v$” would be higher than speed of decaying, generating instability. This chaotic growth of “N” makes no sense according to the second law. We think this error could be explained by the exogeneity of net addition assumed by inflow-driven models.

$$\dot{M}_t = N_t - d_t N_t$$

These accounts were necessarily revised to be adequate for optimization. Considering the saddle point stability, we can conclude that any effort to achieve exponential growth would fail due to substitution. Hence, stock maturity and stability must be deterministically assumed.
References


European Commission. 2015. “Closing the Loop - An EU Action Plan for the Circular Economy - (ANNEX 1).” Communication from the Commission to the European Parliament, the Council, the European Economic and Social Committee and the Committee of the Regions.


(March 13, 2019).


159–67.


